5 CHAPTER SUMMARY

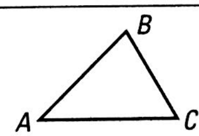
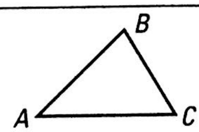
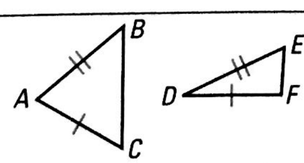
BIG IDEAS

For Your Notebook

Big Idea 1 Using Properties of Special Segments in Triangles

Special segment	*Properties to remember*
Midsegment	Parallel to side opposite it and half the length of side opposite it
Perpendicular bisector	Concurrent at the circumcenter, which is: • equidistant from 3 vertices of △ • center of *circumscribed* circle that passes through 3 vertices of △
Angle bisector	Concurrent at the incenter, which is: • equidistant from 3 sides of △ • center of *inscribed* circle that just touches each side of △
Median (connects vertex to midpoint of opposite side)	Concurrent at the centroid, which is: • located two thirds of the way from vertex to midpoint of opposite side • balancing point of △
Altitude (perpendicular to side of △ through opposite vertex)	Concurrent at the orthocenter Used in finding area: If b is length of any side and h is length of altitude to that side, then $A = \frac{1}{2}bh$.

Big Idea 2 Using Triangle Inequalities to Determine What Triangles are Possible

Sum of lengths of any two sides of a △ is greater than length of third side.		$AB + BC > AC$ $AB + AC > BC$ $BC + AC > AB$
In a △, longest side is opposite largest angle and shortest side is opposite smallest angle.		If $AC > AB > BC$, then $m\angle B > m\angle C > m\angle A$. If $m\angle B > m\angle C > m\angle A$, then $AC > AB > BC$.
If two sides of a △ are ≅ to two sides of another △, then the △ with longer third side also has larger included angle.		If $BC > EF$, then $m\angle A > m\angle D$. If $m\angle A > m\angle D$, then $BC > EF$.

Big Idea 3 Extending Methods for Justifying and Proving Relationships

Coordinate proof uses the coordinate plane and variable coordinates. *Indirect proof* involves assuming the conclusion is false and then showing that the assumption leads to a contradiction.

5 CHAPTER REVIEW

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• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- concurrent, *p. 305*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*
- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

VOCABULARY EXERCISES

1. Copy and complete: A __?__ is a segment, ray, line, or plane that is perpendicular to a segment at its midpoint.
2. **WRITING** *Explain* how to draw a circle that is circumscribed about a triangle. What is the center of the circle called? *Describe* its radius.

In Exercises 3–5, match the term with the correct definition.

3. Incenter — **A.** The point of concurrency of the medians of a triangle
4. Centroid — **B.** The point of concurrency of the angle bisectors of a triangle
5. Orthocenter — **C.** The point of concurrency of the altitudes of a triangle

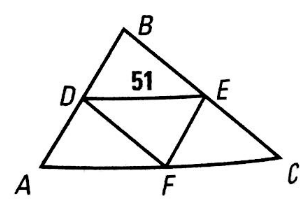
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.

5.1 Midsegment Theorem and Coordinate Proof *pp. 295–301*

EXAMPLE

In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find AC.



By the Midsegment Theorem, $DE = \frac{1}{2}AC$.

So, $AC = 2DE = 2(51) = 102$.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 295, 297 for Exs. 6–8

Use the diagram above where $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.

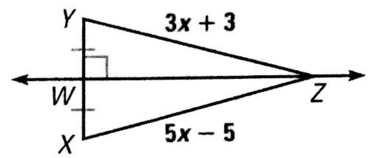
6. If $AB = 72$, find EF.
7. If $DF = 45$, find EC.
8. Graph $\triangle PQR$, with vertices $P(2a, 2b)$, $Q(2a, 0)$, and $O(0, 0)$. Find the coordinates of midpoint S of $\overline{PQ}$ and midpoint T of $\overline{QO}$. Show $\overline{ST} \parallel \overline{PO}$.

5.2 Use Perpendicular Bisectors

pp. 303–309

EXAMPLE

Use the diagram at the right to find XZ.



$\overleftrightarrow{WZ}$ is the perpendicular bisector of $\overline{XY}$.

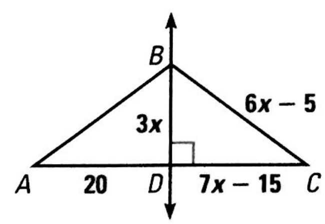
$5x - 5 = 3x + 3$	**By the Perpendicular Bisector Theorem, $ZX = ZY$.**
$x = 4$	**Solve for x.**

▶ So, $XZ = 5x - 5 = 5(4) - 5 = 15$.

EXERCISES

EXAMPLES 1 and 2
on pp. 303–304
for Exs. 9–11

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.



9. What segment lengths are equal?

10. What is the value of x?

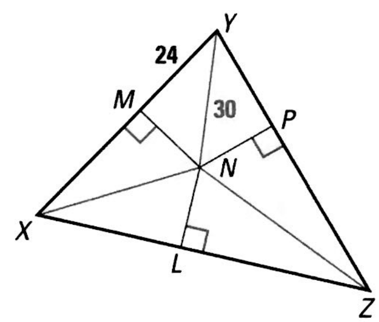
11. Find AB.

5.3 Use Angle Bisectors of Triangles

pp. 310–316

EXAMPLE

In the diagram, N is the incenter of $\triangle XYZ$. Find NL.



Use the Pythagorean Theorem to find NM in $\triangle NMY$.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$30^2 = NM^2 + 24^2$	**Substitute known values.**
$900 = NM^2 + 576$	**Multiply.**
$324 = NM^2$	**Subtract 576 from each side.**
$18 = NM$	**Take positive square root of each side.**

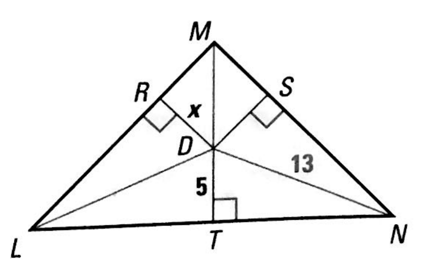
▶ By the Concurrency of Angle Bisectors of a Triangle, the incenter N of $\triangle XYZ$ is equidistant from all three sides of $\triangle XYZ$. So, because $NM = NL$, $NL = 18$.

EXERCISES

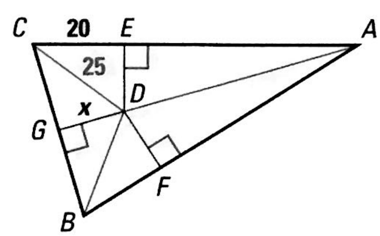
EXAMPLE 4
on p. 312
for Exs. 12–13

Point D is the incenter of the triangle. Find the value of x.

12.



13.



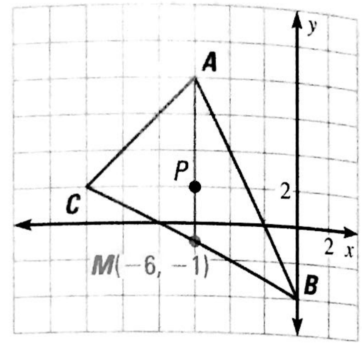
5 CHAPTER REVIEW

5.4 Use Medians and Altitudes

pp. 319–325

EXAMPLE

The vertices of $\triangle ABC$ are $A(-6, 8)$, $B(0, -4)$, and $C(-12, 2)$. Find the coordinates of its centroid P.



Sketch $\triangle ABC$. Then find the midpoint M of $\overline{BC}$ and sketch median $\overline{AM}$.

$$M\left(\frac{-12 + 0}{2}, \frac{2 + (-4)}{2}\right) = M(-6, -1)$$

The centroid is two thirds of the distance from a vertex to the midpoint of the opposite side.

The distance from vertex $A(-6, 8)$ to midpoint $M(-6, -1)$ is $8 - (-1) = 9$ units.

So, the centroid P is $\frac{2}{3}(9) = 6$ units down from A on $\overline{AM}$.

▶ The coordinates of the centroid P are $(-6, 8 - 6)$, or $(-6, 2)$.

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 319–321 for Exs. 14–18

Find the coordinates of the centroid D of $\triangle RST$.

14. $R(-4, 0)$, $S(2, 2)$, $T(2, -2)$

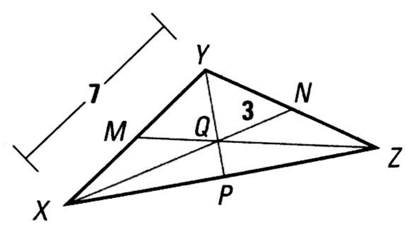
15. $R(-6, 2)$, $S(-2, 6)$, $T(2, 4)$

Point Q is the centroid of $\triangle XYZ$.

16. Find XQ.

17. Find XM.

18. Draw an obtuse $\triangle ABC$. Draw its three altitudes. Then label its orthocenter D.



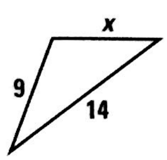
5.5 Use Inequalities in a Triangle

pp. 328–334

EXAMPLE

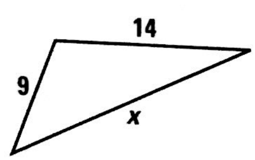
A triangle has one side of length 9 and another of length 14. Describe the possible lengths of the third side.

Let x represent the length of the third side. Draw diagrams and use the Triangle Inequality Theorem to write inequalities involving x.



$x + 9 > 14$

$x > 5$



$9 + 14 > x$

$23 > x$, or $x < 23$

▶ The length of the third side must be greater than 5 and less than 23.

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 328–330 for Exs. 19–24

Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

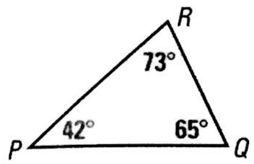
19. 4 inches, 8 inches

20. 6 meters, 9 meters

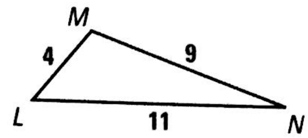
21. 12 feet, 20 feet

List the sides and the angles in order from smallest to largest.

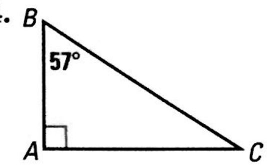
22.



23.



24.

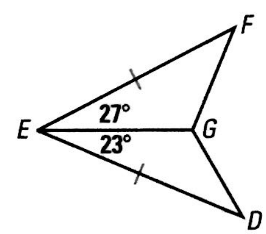


5.6 Inequalities in Two Triangles and Indirect Proof

pp. 335–341

EXAMPLE

How does the length of $\overline{DG}$ compare to the length of $\overline{FG}$?



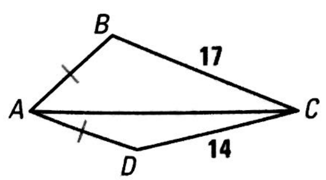
▸ Because 27° > 23°, $m\angle GEF > m\angle GED$. You are given that $\overline{DE} \cong \overline{FE}$ and you know that $\overline{EG} \cong \overline{EG}$. Two sides of $\triangle GEF$ are congruent to two sides of $\triangle GED$ and the included angle is larger so, by the Hinge Theorem, $FG > DG$.

EXERCISES

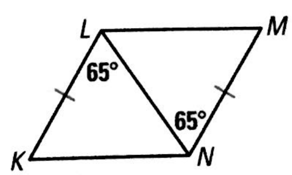
EXAMPLES 1, 3, and 4 on pp. 336–338 for Exs. 25–27

Copy and complete with <, >, or =.

25. $m\angle BAC$ _?_ $m\angle DAC$



26. LM _?_ KN



27. Arrange statements A–D in correct order to write an indirect proof of the statement: *If two lines intersect, then their intersection is exactly one point.*

GIVEN ▸ Intersecting lines m and n

PROVE ▸ The intersection of lines m and n is exactly one point.

A. But this contradicts Postulate 5, which states that through any two points there is exactly one line.

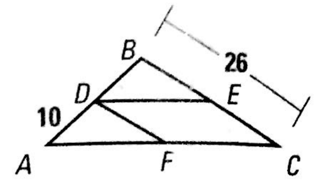
B. Then there are two lines (m and n) through points P and Q.

C. Assume that there are two points, P and Q, where m and n intersect.

D. It is false that m and n can intersect in two points, so they must intersect in exactly one point.

5 CHAPTER TEST

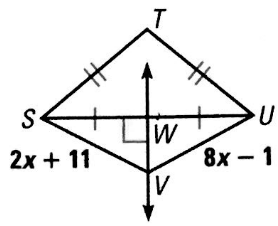
Two midsegments of $\triangle ABC$ are $\overline{DE}$ and $\overline{DF}$.



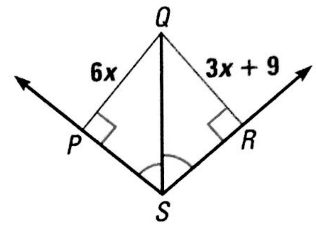
1. Find DB.
2. Find DF.
3. What can you conclude about $\overline{EF}$?

Find the value of x. *Explain* your reasoning.

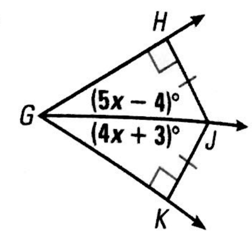
4.



5.

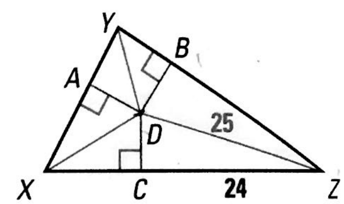


6.

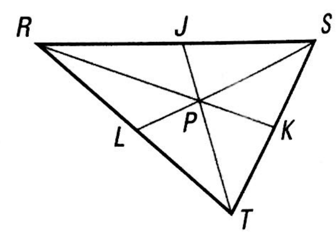


7. In Exercise 4, is point T on the perpendicular bisector of $\overline{SU}$? *Explain.*

8. In the diagram at the right, the angle bisectors of $\triangle XYZ$ meet at point D. Find DB.



In the diagram at the right, P is the centroid of $\triangle RST$.

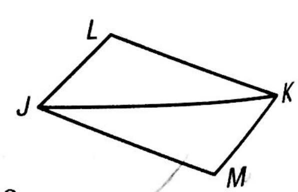


9. If $LS = 36$, find PL and PS.
10. If $TP = 20$, find TJ and PJ.
11. If $JR = 25$, find JS and RS.

12. Is it possible to construct a triangle with side lengths 9, 12, and 22? If not, *explain* why not.

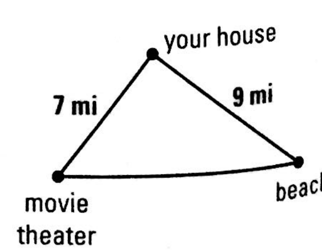
13. In $\triangle ABC$, $AB = 36$, $BC = 18$, and $AC = 22$. Sketch and label the triangle. List the angles in order from smallest to largest.

In the diagram for Exercises 14 and 15, $JL = MK$.



14. If $m\angle JKM > m\angle LJK$, which is longer, $\overline{LK}$ or $\overline{MJ}$? *Explain.*
15. If $MJ < LK$, which is larger, $\angle LJK$ or $\angle JKM$? *Explain.*
16. Write a temporary assumption you could make to prove the conclusion indirectly: *If $RS + ST \neq 12$ and $ST = 5$, then $RS \neq 7$.*

Use the diagram in Exercises 17 and 18.



17. *Describe* the range of possible distances from the beach to the movie theater.
18. A market is the same distance from your house, the movie theater, and the beach. Copy the diagram and locate the market.

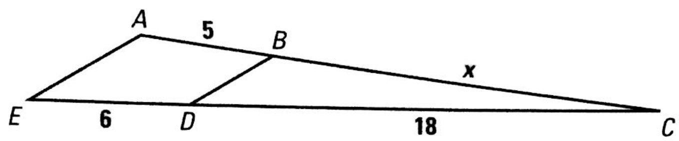
BIG IDEAS

For Your Notebook

Big Idea 1

Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.



For example, in the diagram above, suppose you know that $\frac{AB}{BC} = \frac{ED}{DC}$. Then you can write any of the following relationships.

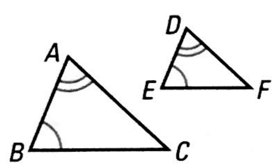
$\frac{5}{x} = \frac{6}{18}$ $\quad 5 \cdot 18 = 6x \quad$ $\frac{x}{5} = \frac{18}{6}$ $\quad \frac{5}{6} = \frac{x}{18} \quad$ $\frac{5+x}{x} = \frac{6+18}{18}$

Big Idea 2

Showing that Triangles are Similar

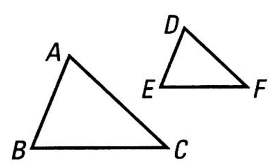
You learned three ways to prove two triangles are similar.

AA Similarity Postulate



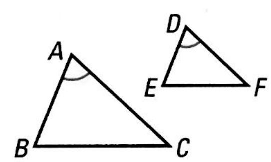
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

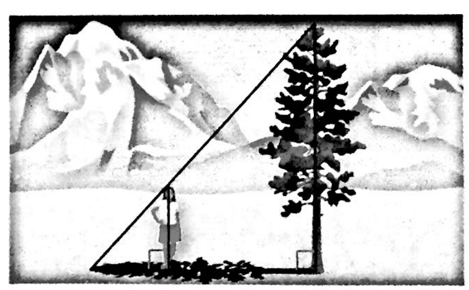


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Big Idea 3

Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.



Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.

6 CHAPTER REVIEW

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- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- ratio, *p. 356*
- proportion, *p. 358* means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

VOCABULARY EXERCISES

Copy and complete the statement.

1. A _?_ is a transformation in which the original figure and its image are similar.
2. If $\triangle PQR \sim \triangle XYZ$, then $\frac{PQ}{XY} = \frac{?}{YZ} = \frac{?}{?}$.
3. **WRITING** *Describe* the relationship between a ratio and a proportion. Give an example of each.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Ratios, Proportions, and the Geometric Mean *pp. 356–363*

EXAMPLE

The measures of the angles in $\triangle ABC$ are in the extended ratio of 3 : 4 : 5. Find the measures of the angles.

Use the extended ratio of 3 : 4 : 5 to label the angle measures as $3x°$, $4x°$, and $5x°$.

$3x° + 4x° + 5x° = 180°$	**Triangle Sum Theorem**
$12x = 180$	**Combine like terms.**
$x = 15$	**Divide each side by 12.**

So, the angle measures are $3(15°) = 45°$, $4(15°) = 60°$, and $5(15°) = 75°$.

EXERCISES

EXAMPLES 1, 3, and 6 on pp. 356–359 for Exs. 4–6

4. The length of a rectangle is 20 meters and the width is 15 meters. Find the ratio of the width to the length of the rectangle. Then simplify the ratio.
5. The measures of the angles in $\triangle UVW$ are in the extended ratio of 1 : 1 : 2. Find the measures of the angles.
6. Find the geometric mean of 8 and 12.

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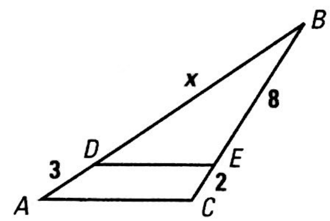
Chapter Review Practice

6.2 Use Proportions to Solve Geometry Problems

pp. 364–370

EXAMPLE

In the diagram, $\frac{BA}{DA} = \frac{BC}{EC}$. Find *BD*.



$\frac{x+3}{3} = \frac{8+2}{2}$ **Substitution Property of Equality**

$2x + 6 = 30$ **Cross Products Property**

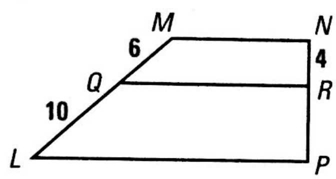
$x = 12$ **Solve for *x*.**

EXERCISES

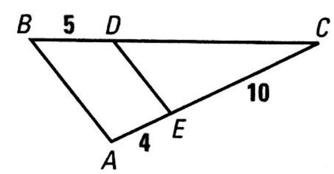
EXAMPLE 2 on p. 365 for Exs. 7–8

Use the diagram and the given information to find the unknown length.

7. Given $\frac{RN}{RP} = \frac{QM}{QL}$, find *RP*.



8. Given $\frac{CD}{DB} = \frac{CE}{EA}$, find *CD*.

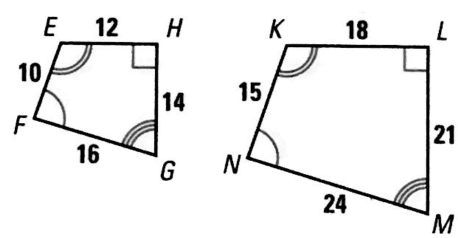


6.3 Use Similar Polygons

pp. 372–379

EXAMPLE

In the diagram, *EHGF* ~ *KLMN*. Find the scale factor.



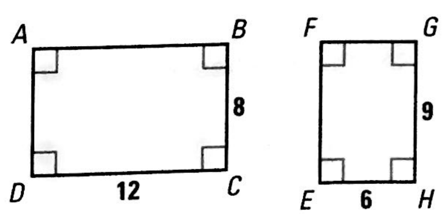
From the diagram, you can see that $\overline{EH}$ and $\overline{KL}$ correspond. So, the scale factor of *EHGF* to *KLMN* is $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$.

EXERCISES

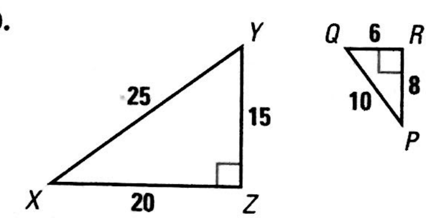
EXAMPLES 2 and 4 on pp. 373–374 for Exs. 9–11

In Exercises 9 and 10, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

9.



10.



11. **POSTERS** Two similar posters have a scale factor of 4 : 5. The large poster's perimeter is 85 inches. Find the small poster's perimeter.

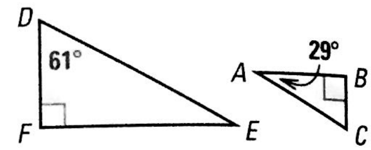
6 CHAPTER REVIEW

6.4 Prove Triangles Similar by AA

pp. 381–387

EXAMPLE

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



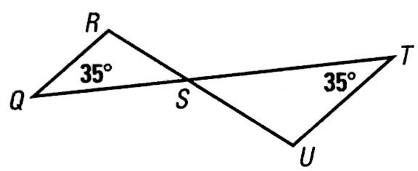
Because they are right angles, $\angle F \cong \angle B$. By the Triangle Sum Theorem, $61° + 90° + m\angle E = 180°$, so $m\angle E = 29°$ and $\angle E \cong \angle A$. Then, two angles of $\triangle DFE$ are congruent to two angles of $\triangle CBA$. So, $\triangle DFE \sim \triangle CBA$.

EXERCISES

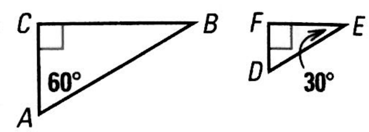
EXAMPLES 2 and 3 on pp. 382–383 for Exs. 12–14

Use the AA Similarity Postulate to show that the triangles are similar.

12.



13.



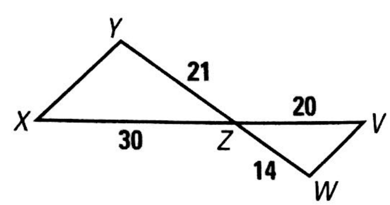
14. CELL TOWER A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

6.5 Prove Triangles Similar by SSS and SAS

pp. 388–395

EXAMPLE

Show that the triangles are similar.



Notice that the lengths of two pairs of corresponding sides are proportional.

$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \qquad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$

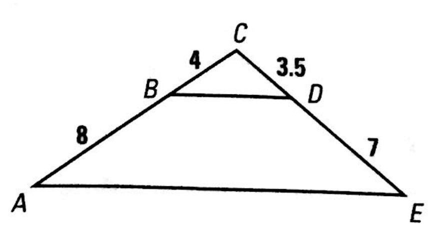
The included angles for these sides, $\angle XZY$ and $\angle VZW$, are vertical angles, so $\angle XZY \cong \angle VZW$. Then $\triangle XYZ \sim \triangle VWZ$ by the SAS Similarity Theorem.

EXERCISES

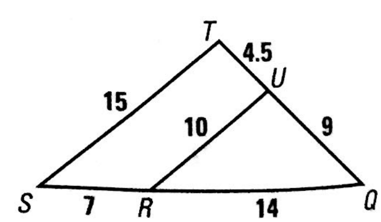
EXAMPLE 4 on p. 391 for Exs. 15–16

Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar.

15.



16.



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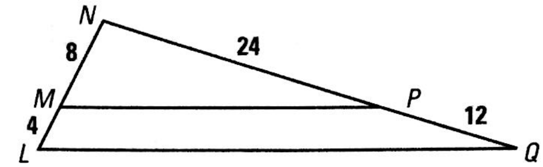
Chapter Review Practice

6.6 Use Proportionality Theorems

pp. 397–403

EXAMPLE

Determine whether $\overline{MP} \parallel \overline{LQ}$.



Begin by finding and simplifying ratios of lengths determined by $\overline{MP}$.

$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1}$ $\qquad$ $\frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$

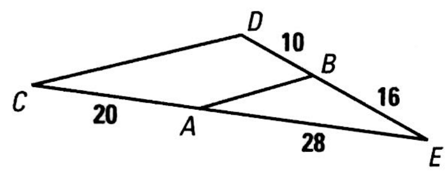
Because $\frac{NM}{ML} = \frac{NP}{PQ}$, $\overline{MP}$ is parallel to $\overline{LQ}$ by Theorem 6.5, the Triangle Proportionality Converse.

EXERCISES

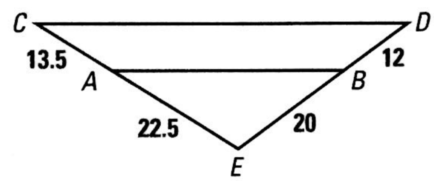
EXAMPLE 2
on p. 398
for Exs. 17–18

Use the given information to determine whether $\overline{AB} \parallel \overline{CD}$.

17.



18.



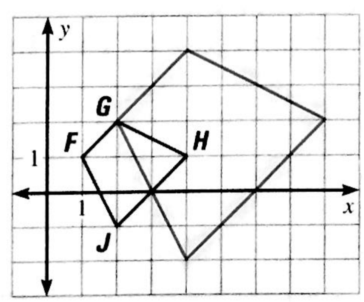
6.7 Perform Similarity Transformations

pp. 409–415

EXAMPLE

Draw a dilation of quadrilateral *FGHJ* with vertices $F(1, 1)$, $G(2, 2)$, $H(4, 1)$, and $J(2, -1)$. Use a scale factor of 2.

First draw *FGHJ*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.



$(x, y) \rightarrow (2x, 2y)$

$F(1, 1) \rightarrow (2, 2)$

$G(2, 2) \rightarrow (4, 4)$

$H(4, 1) \rightarrow (8, 2)$

$J(2, -1) \rightarrow (4, -2)$

EXERCISES

EXAMPLE 1
on p. 409
for Exs. 19–21

Draw a dilation of the polygon with the given vertices using the given scale factor *k*.

19. $T(0, 8)$, $U(6, 0)$, $V(0, 0)$; $k = \frac{3}{2}$

20. $A(6, 0)$, $B(3, 9)$, $C(0, 0)$, $D(3, 1)$; $k = 4$

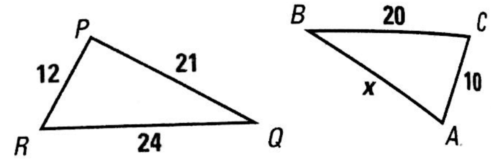
21. $P(8, 2)$, $Q(4, 0)$, $R(3, 1)$, $S(6, 4)$; $k = 0.5$

6 CHAPTER TEST

Solve the proportion.

1. $\frac{6}{x} = \frac{9}{24}$ **2.** $\frac{5}{4} = \frac{y-5}{12}$ **3.** $\frac{3-2b}{4} = \frac{3}{2}$ **4.** $\frac{7}{2a+8} = \frac{1}{a-1}$

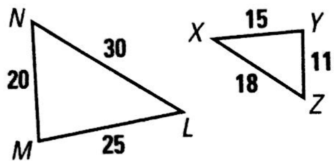
In Exercises 5–7, use the diagram where $\triangle PQR \sim \triangle ABC$.



5. List all pairs of congruent angles.
6. Write the ratios of the corresponding sides in a statement of proportionality.
7. Find the value of x.

Determine whether the triangles are similar. If so, write a similarity statement and the postulate or theorem that justifies your answer.

8.



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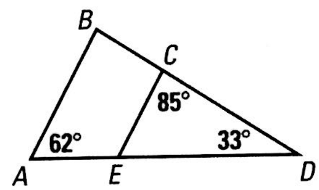

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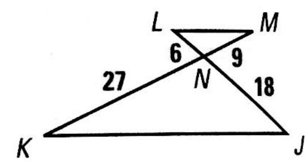
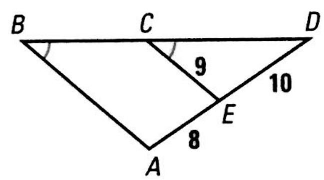


In Exercises 11–13, find the length of $\overline{AB}$.

11.



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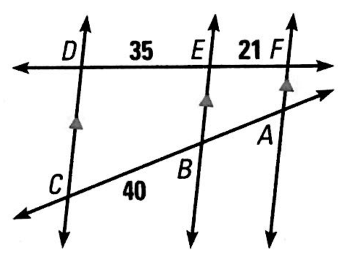

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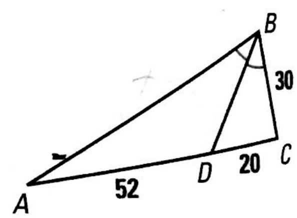
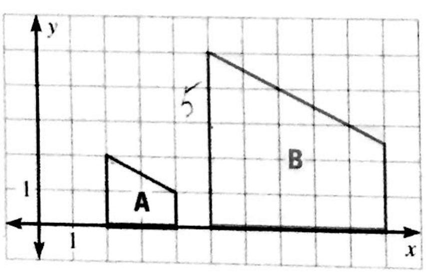
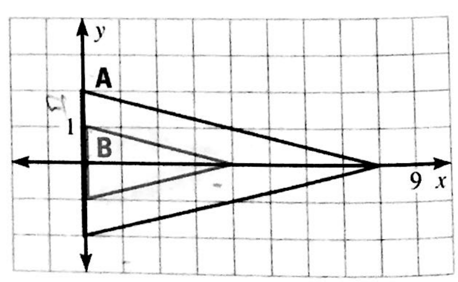


Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.

14.



15.



16. SCALE MODEL You are making a scale model of your school's baseball diamond as part of an art project. The distance between two consecutive bases is 90 feet. If you use a scale factor of $\frac{1}{180}$ to build your model, what will be the distance around the bases on your model?