7 CHAPTER SUMMARY

BIG IDEAS

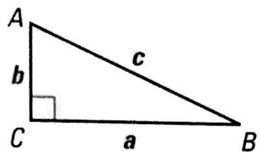
For Your Notebook

Big Idea 1

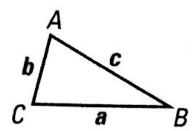
Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

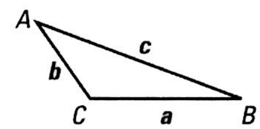
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.



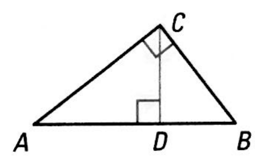
If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2

Using Special Relationships in Right Triangles

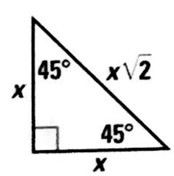
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



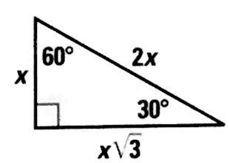
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg • $\sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

Big Idea 3

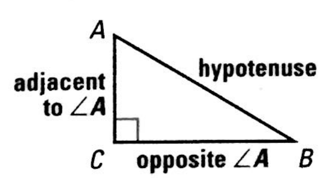
Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC}$ $\qquad \tan^{-1} \frac{BC}{AC} = m\angle A$

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB}$ $\qquad \sin^{-1} \frac{BC}{AB} = m\angle A$

$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB}$ $\qquad \cos^{-1} \frac{AC}{AB} = m\angle A$



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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation __?__.
2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?
3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation.

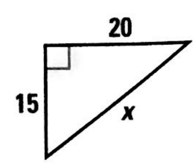
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem *pp. 433–439*

EXAMPLE

Find the value of x.



Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

$x^2 = 15^2 + 20^2$ **Substitute.**

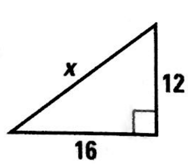
$x^2 = 625$ **Simplify.**

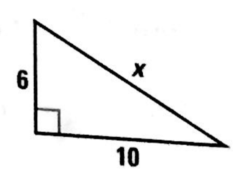
$x = 25$ **Find the positive square root.**

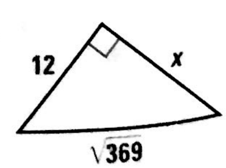
EXERCISES

EXAMPLES 1 and 2 on pp. 433–434 for Exs. 4–6

Find the unknown side length x.

4. 

5. 

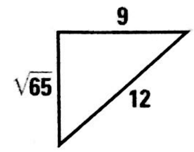
6. 

7.2 Use the Converse of the Pythagorean Theorem *pp. 441–447*

EXAMPLE

Tell whether the given triangle is a right triangle.

Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.



$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$

$144 \stackrel{?}{=} 65 + 81$

$144 < 146$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

KAMPLE 2
p. 442
r Exs. 7–12

Classify the triangle formed by the side lengths as *acute, right,* or *obtuse.*

7. 6, 8, 9 **8.** 4, 2, 5 **9.** 10, $2\sqrt{2}$, $6\sqrt{3}$

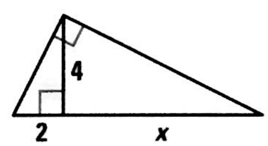
10. 15, 20, 15 **11.** 3, 3, $3\sqrt{2}$ **12.** 13, 18, $3\sqrt{55}$

7.3 Use Similar Right Triangles *pp. 449–456*

EXAMPLE

Find the value of *x*.

By Theorem 7.6, you know that 4 is the geometric mean of x and 2.



$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

$2x = 16$ **Cross Products Property**

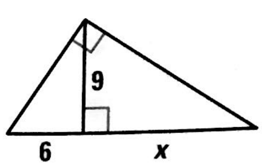
$x = 8$ **Divide.**

EXERCISES

MPLES
d 3
p. 450–451
xs. 13–18

Find the value of *x*.

13.



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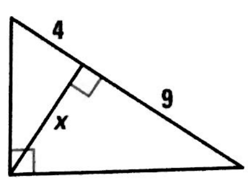

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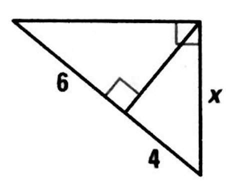

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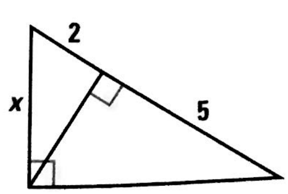
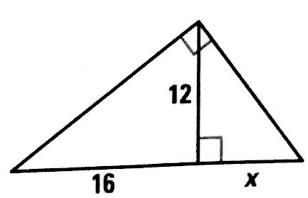
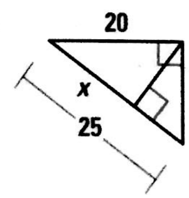


17.



18.



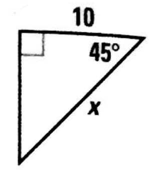
7 CHAPTER REVIEW

7.4 Special Right Triangles

pp. 457–464

EXAMPLE

Find the length of the hypotenuse.



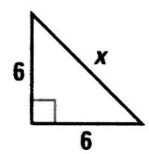
By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

hypotenuse $=$ leg $\cdot \sqrt{2}$	**45°-45°-90° Triangle Theorem**
$x = 10\sqrt{2}$	**Substitute.**

EXERCISES

EXAMPLES 1, 2, and 5 on pp. 457–459 for Exs. 19–21

Find the value of x. Write your answer in simplest radical form.

19.


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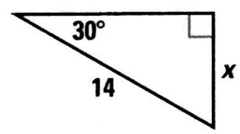

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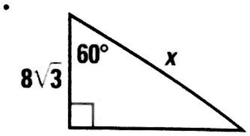


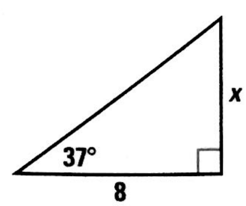
7.5 Apply the Tangent Ratio

pp. 466–472

EXAMPLE

Find the value of x.

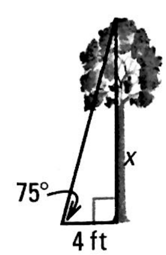
$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio for tangent of 37°.**
$\tan 37° = \frac{x}{8}$	**Substitute.**
$8 \cdot \tan 37° = x$	**Multiply each side by 8.**
$6 \approx x$	**Use a calculator to simplify.**



EXERCISES

EXAMPLE 2 on p. 467 for Exs. 22–26

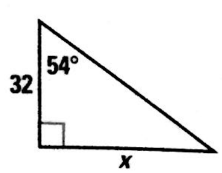
In Exercises 22 and 23, use the diagram.

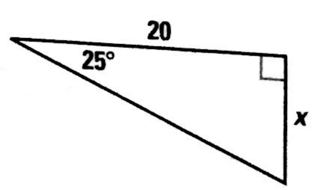


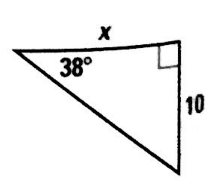
22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

23. In Exercise 22, how tall is the tree if the angle is 55°?

Find the value of x to the nearest tenth.

24.


25.


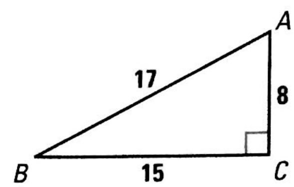
26.


7.6 Apply the Sine and Cosine Ratios

pp. 473–480

EXAMPLE

Find sin A and sin B.



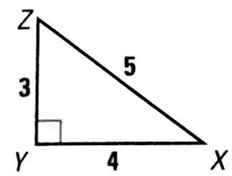
$$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$$

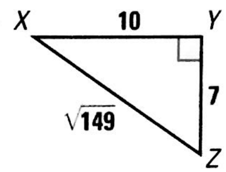
$$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$$

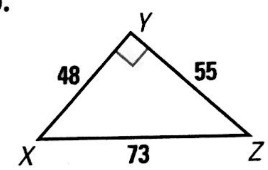
EXERCISES

EXAMPLES 1 and 2 on pp. 473–474 for Exs. 27–29

Find sin X and cos X. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

27.


28.


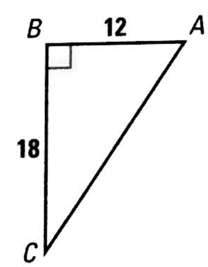
29.


7.7 Solve Right Triangles

pp. 483–489

EXAMPLE

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

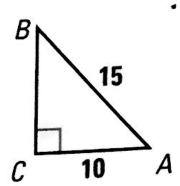
$\tan^{-1} 1.5 \approx 56.3099324\ldots$

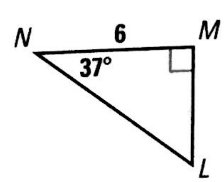
So, the measure of $\angle A$ is approximately 56.3°.

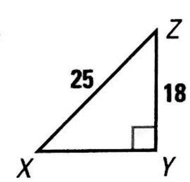
EXERCISES

EXAMPLE 3 on p. 484 for Exs. 30–33

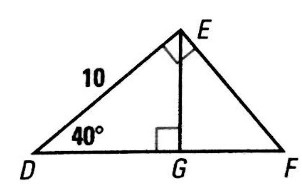
Solve the right triangle. Round decimal answers to the nearest tenth.

30.


31.


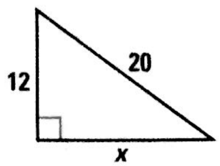
32.


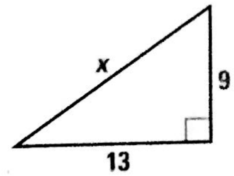
33. Find the measures of $\angle GED$, $\angle GEF$, and $\angle EFG$. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$.

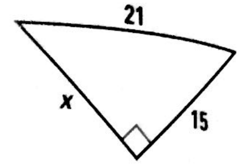


7 CHAPTER TEST

Find the value of x. Write your answer in simplest radical form.

1. 

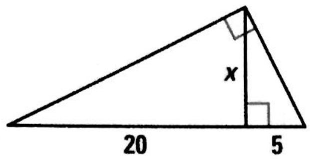
2. 

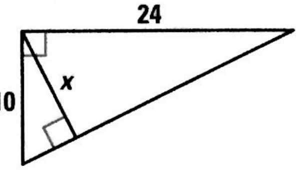
3. 

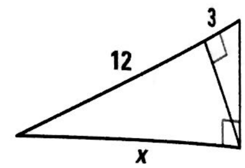
Classify the triangle as *acute*, *right*, or *obtuse*.

4. $5, 15, 5\sqrt{10}$
5. 4.3, 6.7, 8.2
6. 5, 7, 8

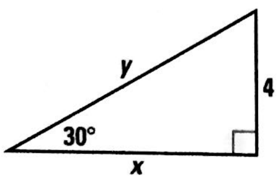
Find the value of x. Round decimal answers to the nearest tenth.

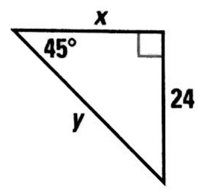
7. 

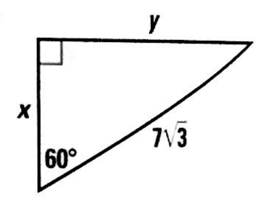
8. 

9. 

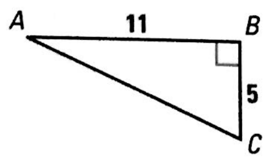
Find the value of each variable. Write your answer in simplest radical form.

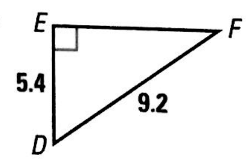
10. 

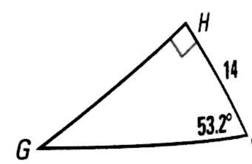
11. 

12. 

Solve the right triangle. Round decimal answers to the nearest tenth.

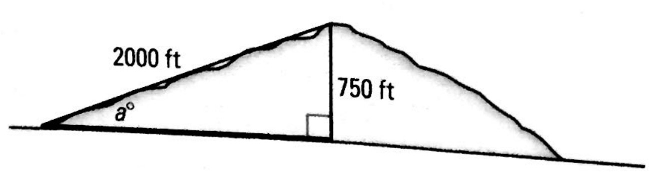
13. 

14. 

15. 

16. **FLAGPOLE** Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole.

17. **HILLS** The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill?



8 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Using Angle Relationships in Polygons

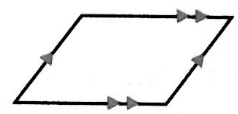
You can use theorems about the interior and exterior angles of convex polygons to solve problems.

Polygon Interior Angles Theorem	Polygon Exterior Angles Theorem
The sum of the interior angle measures of a convex n-gon is $(n - 2) \cdot 180°$.	The sum of the exterior angle measures of a convex n-gon is $360°$.

Big Idea 2

Using Properties of Parallelograms

By definition, a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Other properties of parallelograms:



- Opposite sides are congruent.
- Diagonals bisect each other.
- Opposite angles are congruent.
- Consecutive angles are supplementary.

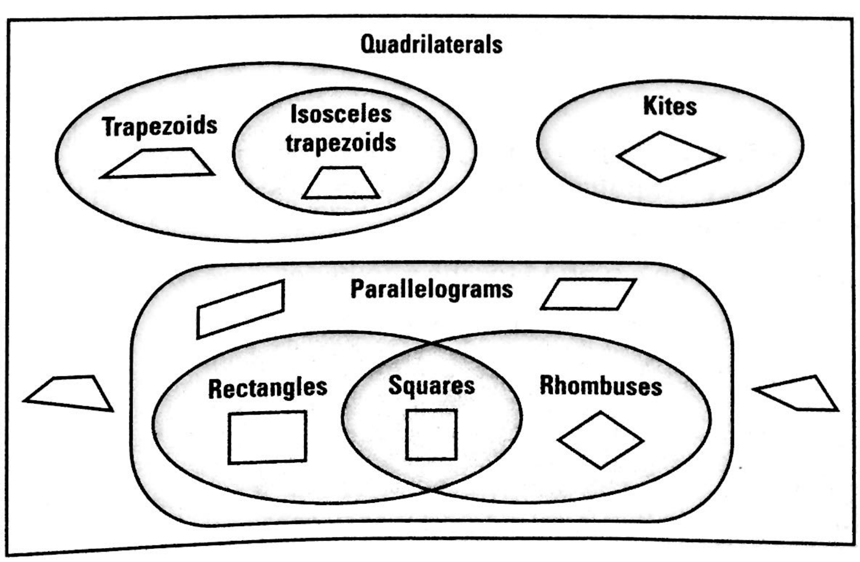
Ways to show that a quadrilateral is a parallelogram:

- Show both pairs of opposite sides are parallel.
- Show both pairs of opposite sides or opposite angles are congruent.
- Show one pair of opposite sides are congruent and parallel.
- Show the diagonals bisect each other.

Big Idea 3

Classifying Quadrilaterals by Their Properties

Special quadrilaterals can be classified by their properties. In a parallelogram, both pairs of opposite sides are parallel. In a trapezoid, only one pair of sides are parallel. A kite has two pairs of consecutive congruent sides, but opposite sides are not congruent.



8 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

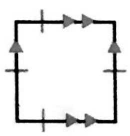
- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
- bases of a trapezoid, *p. 542*
- base angles of a trapezoid, *p. 542*
- legs of a trapezoid, *p. 542*
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

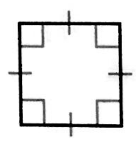
VOCABULARY EXERCISES

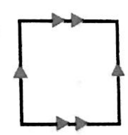
In Exercises 1 and 2, copy and complete the statement.

1. The _?_ of a trapezoid is parallel to the bases.
2. A(n) _?_ of a polygon is a segment whose endpoints are nonconsecutive vertices.
3. **WRITING** *Describe* the different ways you can show that a trapezoid is an isosceles trapezoid.

In Exercises 4–6, match the figure with the most specific name.

4. 

5. 

6. 

A. Square **B.** Parallelogram **C.** Rhombus

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

8.1 Find Angle Measures in Polygons

pp. 507–513

EXAMPLE

The sum of the measures of the interior angles of a convex regular polygon is 1080°. Classify the polygon by the number of sides. What is the measure of each interior angle?

Write and solve an equation for the number of sides n.

$(n - 2) \cdot 180° = 1080°$ **Polygon Interior Angles Theorem**

$n = 8$ **Solve for n.**

The polygon has 8 sides, so it is an octagon.

A regular octagon has 8 congruent interior angles, so divide to find the measure of each angle: $1080° \div 8 = 135°$. The measure of each interior angle is 135°.

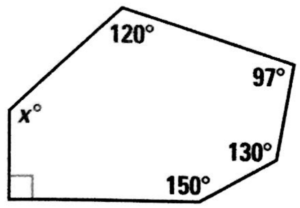
EXAMPLES 2, 3, 4, and 5 on pp. 508–510 for Exs. 7–11

EXERCISES

7. The sum of the measures of the interior angles of a convex regular polygon is 3960°. Classify the polygon by the number of sides. What is the measure of each interior angle?

In Exercises 8–10, find the value of x.

8.



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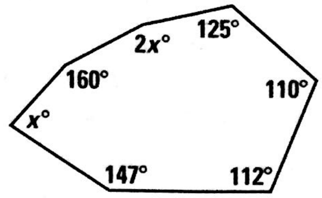
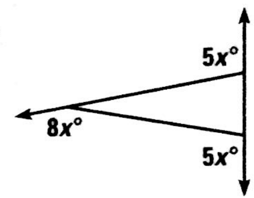


10.

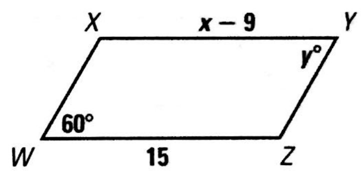


11. In a regular nonagon, the exterior angles are all congruent. What is the measure of one of the exterior angles? *Explain.*

8.2 Use Properties of Parallelograms *pp. 515–521*

EXAMPLE

Quadrilateral *WXYZ* is a parallelogram. Find the values of x and y.



To find the value of x, apply Theorem 8.3.

$XY = WZ$	**Opposite sides of a ▱ are ≅.**
$x - 9 = 15$	**Substitute.**
$x = 24$	**Add 9 to each side.**

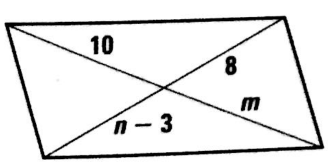
By Theorem 8.4, $\angle W \cong \angle Y$, or $m\angle W = m\angle Y$. So, $y = 60$.

EXAMPLES 1, 2, and 3 on pp. 515, 517 for Exs. 12–17

EXERCISES

Find the value of each variable in the parallelogram.

12.



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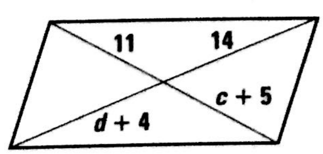

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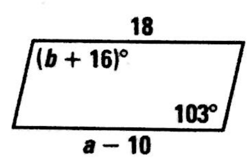


15. In ▱*PQRS*, $PQ = 5$ centimeters, $QR = 10$ centimeters, and $m\angle PQR = 36°$. Sketch *PQRS*. Find and label all of its side lengths and interior angle measures.

16. The perimeter of ▱*EFGH* is 16 inches. If *EF* is 5 inches, find the lengths of all the other sides of *EFGH*. *Explain* your reasoning.

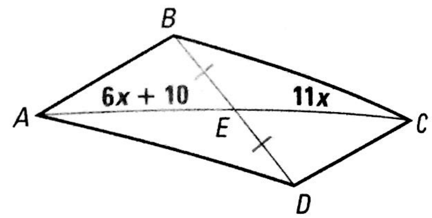
17. In ▱*JKLM*, the ratio of the measure of $\angle J$ to the measure of $\angle M$ is 5 : 4. Find $m\angle J$ and $m\angle M$. *Explain* your reasoning.

8.3 Show that a Quadrilateral is a Parallelogram

pp. 522–529

EXAMPLE

For what value of x is quadrilateral $ABCD$ a parallelogram?



If the diagonals bisect each other, then $ABCD$ is a parallelogram. The diagram shows that $\overline{BE} \cong \overline{DE}$. You need to find the value of x that makes $\overline{AE} \cong \overline{CE}$.

$AE = CE$	**Set the segment lengths equal.**
$6x + 10 = 11x$	**Substitute expressions for the lengths.**
$x = 2$	**Solve for x.**

When $x = 2$, $AE = 6(2) + 10 = 22$ and $CE = 11(2) = 22$. So, $\overline{AE} \cong \overline{CE}$.

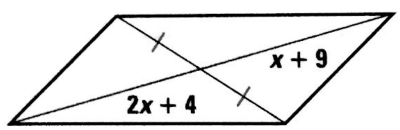
Quadrilateral $ABCD$ is a parallelogram when $x = 2$.

EXERCISES

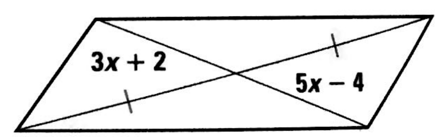
EXAMPLE 3 on p. 524 for Exs. 18–19

For what value of x is the quadrilateral a parallelogram?

18.



19.

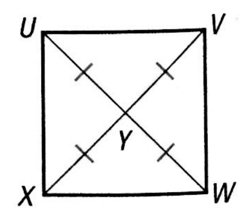


8.4 Properties of Rhombuses, Rectangles, and Squares

pp. 533–540

EXAMPLE

Classify the special quadrilateral.



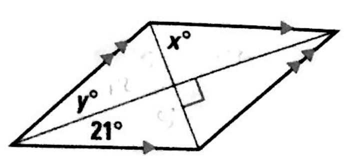
In quadrilateral $UVWX$, the diagonals bisect each other. So, $UVWX$ is a parallelogram. Also, $\overline{UY} \cong \overline{VY} \cong \overline{WY} \cong \overline{XY}$. So, $UY + YW = VY + XY$. Because $UY + YW = UW$, and $VY + XY = VX$, you can conclude that $\overline{UW} \cong \overline{VX}$. By Theorem 8.13, $UVWX$ is a rectangle.

EXERCISES

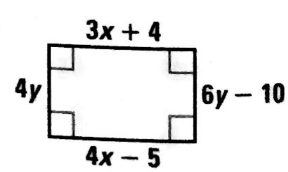
EXAMPLES 2 and 3 on pp. 534–535 for Exs. 20–22

Classify the special quadrilateral. Then find the values of x and y.

20.



21.



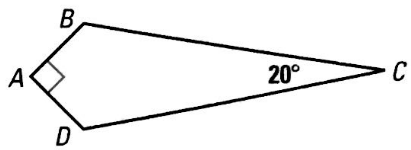
22. The diagonals of a rhombus are 10 centimeters and 24 centimeters. Find the length of a side. *Explain.*

8.5 Use Properties of Trapezoids and Kites

pp. 542–549

EXAMPLE

Quadrilateral *ABCD* is a kite. Find $m\angle B$ and $m\angle D$.



A kite has exactly one pair of congruent opposite angles. Because $\angle A \not\cong \angle C$, $\angle B$ and $\angle D$ must be congruent. Write and solve an equation.

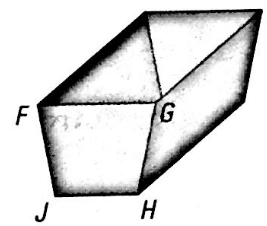
$90° + 20° + m\angle B + m\angle D = 360°$	**Corollary to Theorem 8.1**
$110° + m\angle B° + m\angle D = 360°$	**Combine like terms.**
$m\angle B + m\angle D = 250°$	**Subtract 110° from each side.**

Because $\angle B \cong \angle D$, you can substitute $m\angle B$ for $m\angle D$ in the last equation. Then $m\angle B + m\angle B = 250°$, and $m\angle B = m\angle D = 125°$.

EXERCISES

EXAMPLES 2 and 3 on pp. 543–544 for Exs. 20–22

In Exercises 23 and 24, use the diagram of a recycling container. One end of the container is an isosceles trapezoid with $\overline{FG} \parallel \overline{JH}$ and $m\angle F = 79°$.



23. Find $m\angle G$, $m\angle H$, and $m\angle J$.

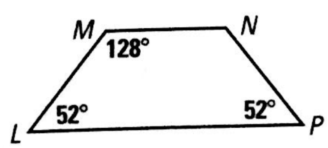
24. Copy trapezoid *FGHJ* and sketch its midsegment. If the midsegment is 16.5 inches long and $\overline{FG}$ is 19 inches long, find *JH*.

8.6 Identify Special Quadrilaterals

pp. 552–557

EXAMPLE

Give the most specific name for quadrilateral *LMNP*.



In *LMNP*, $\angle L$ and $\angle M$ are supplementary, but $\angle L$ and $\angle P$ are not. So, $\overline{MN} \parallel \overline{LP}$, but $\overline{LM}$ is not parallel to $\overline{NP}$. By definition, *LMNP* is a trapezoid.

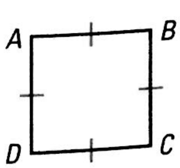
Also, $\angle L$ and $\angle P$ are a pair of base angles and $\angle L \cong \angle P$. So, *LMNP* is an isosceles trapezoid by Theorem 8.15.

EXERCISES

EXAMPLE 2 on p. 553 for Exs. 25–28

Give the most specific name for the quadrilateral. *Explain* your reasoning.

25.



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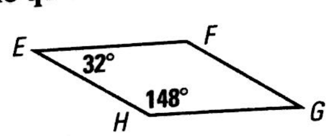

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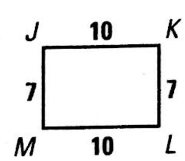
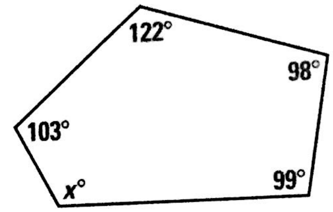


28. In quadrilateral *RSTU*, $\angle R$, $\angle T$, and $\angle U$ are right angles, and $RS = ST$. What is the most specific name for quadrilateral *RSTU*? *Explain.*

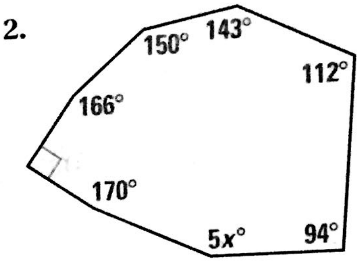
8 CHAPTER TEST

Find the value of x.

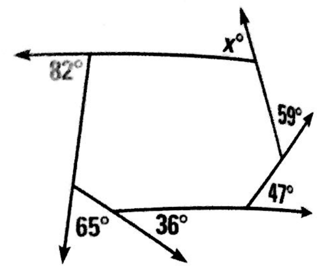
1.



2.



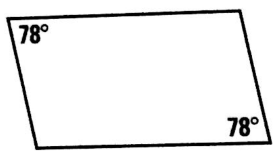
3.



4. In ▱$EFGH$, $m\angle F$ is 40° greater than $m\angle G$. Sketch ▱$EFGH$ and label each angle with its correct angle measure. *Explain* your reasoning.

Are you given enough information to determine whether the quadrilateral is a parallelogram? *Explain* your reasoning.

5.



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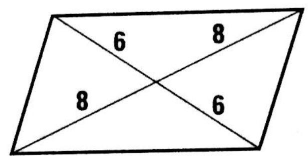

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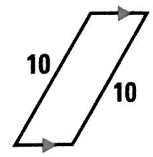


In Exercises 8–11, list each type of quadrilateral—*parallelogram*, *rectangle*, *rhombus*, and *square*—for which the statement is always true.

8. It is equilateral.

9. Its interior angles are all right angles.

10. The diagonals are congruent.

11. Opposite sides are parallel.

12. The vertices of quadrilateral $PQRS$ are $P(-2, 0)$, $Q(0, 3)$, $R(6, -1)$, and $S(1, -2)$. Draw $PQRS$ in a coordinate plane. Show that it is a trapezoid.

13. One side of a quadrilateral $JKLM$ is longer than another side.

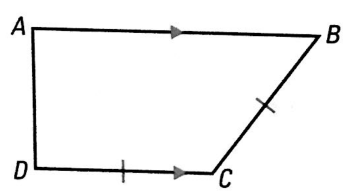
a. Suppose $JKLM$ is an isosceles trapezoid. In a coordinate plane, find possible coordinates for the vertices of $JKLM$. *Justify* your answer.

b. Suppose $JKLM$ is a kite. In a coordinate plane, find possible coordinates for the vertices of $JKLM$. *Justify* your answer.

c. Name other special quadrilaterals that $JKLM$ could be.

Give the most specific name for the quadrilateral. *Explain* your reasoning.

14.



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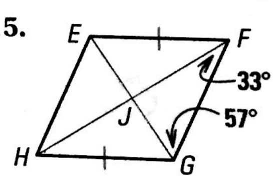

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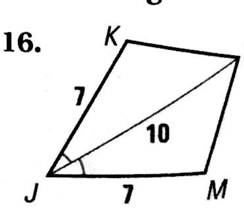

17. In trapezoid $WXYZ$, $\overline{WX} \parallel \overline{YZ}$, and $YZ = 4.25$ centimeters. The midsegment of trapezoid $WXYZ$ is 2.75 centimeters long. Find WX.

18. In ▱$RSTU$, $\overline{RS}$ is 3 centimeters shorter than $\overline{ST}$. The perimeter of ▱$RSTU$ is 42 centimeters. Find RS and ST.