4-2 Transformations on the Coordinate Plane

What You'll Learn

- Transform figures by using reflections, translations, dilations, and rotations.
- Transform figures on a coordinate plane by using reflections, translations, dilations, and rotations.

Vocabulary

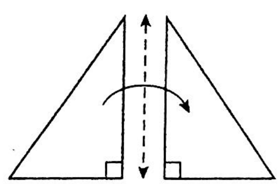
- transformation
- preimage
- image
- reflection
- translation
- dilation
- rotation

How are transformations used in computer graphics?

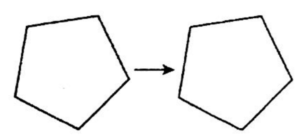
Computer programs can create movements that mimic real-life situations. A new CD-ROM-based flight simulator replicates an actual flight experience so closely that the U.S. Navy is using it for all of their student aviators. The movements of the on-screen graphics are accomplished by using mathematical transformations.

TRANSFORM FIGURES **Transformations** are movements of geometric figures. The **preimage** is the position of the figure before the transformation, and the **image** is the position of the figure after the transformation.

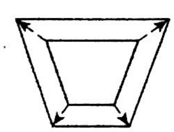
reflection
a figure is flipped over a line



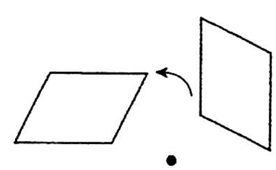
translation
a figure is slid in any direction



dilation
a figure is enlarged or reduced



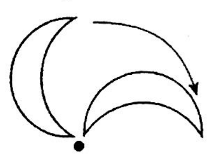
rotation
a figure is turned around a point



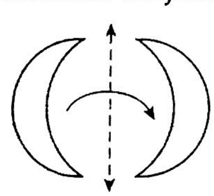
Example 1 *Identify Transformations*

Identify each transformation as a *reflection, translation, dilation,* or *rotation.*

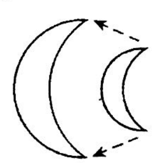
a.



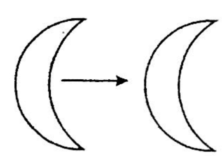
b.



c.



d.



a. The figure has been turned around a point. This is a rotation.

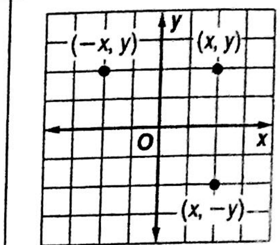
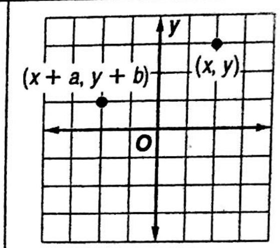
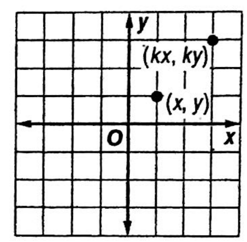
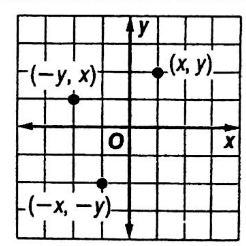
b. The figure has been flipped over a line. This is a reflection.

c. The figure has been increased in size. This is a dilation.

d. The figure has been shifted horizontally to the right. This is a translation.

TRANSFORM FIGURES ON THE COORDINATE PLANE You can perform transformations on a coordinate plane by changing the coordinates of the points on a figure. The points on the translated figure are indicated by the prime symbol ′ to distinguish them from the original points.

Key Concept — *Transformations on the Coordinate Plane*

Name	Words	Symbols	Model
Reflection	To reflect a point over the x-axis, multiply the y-coordinate by -1. To reflect a point over the y-axis, multiply the x-coordinate by -1.	reflection over x-axis: $(x, y) \rightarrow (x, -y)$ reflection over y-axis: $(x, y) \rightarrow (-x, y)$	
Translation	To translate a point by an ordered pair (a, b), add a to the x-coordinate and b to the y-coordinate.	$(x, y) \rightarrow (x + a, y + b)$	
Dilation	To dilate a figure by a scale factor k, multiply both coordinates by k. If $k > 1$, the figure is enlarged. If $0 < k < 1$, the figure is reduced.	$(x, y) \rightarrow (kx, ky)$	
Rotation 270° counter clock. or 90° clockwise $(x,y) \rightarrow (y,-x)$ switch new 2ND mult. by −1.	To rotate a figure 90° *counterclockwise* about the origin, switch the coordinates of each point and then multiply the new first coordinate by -1. To rotate a figure 180° about the origin, multiply both coordinates of each point by -1.	90° rotation: $(x, y) \rightarrow (-y, x)$ 180° rotation: $(x, y) \rightarrow (-x, -y)$	

Study Tip

Reading Math
The *vertices* of a polygon are the endpoints of the angles.

Example 2 *Reflection*

A parallelogram has vertices $A(-4, 3)$, $B(1, 3)$, $C(0, 1)$, and $D(-5, 1)$.

a. Parallelogram $ABCD$ is reflected over the x-axis. Find the coordinates of the vertices of the image.

To reflect the figure over the x-axis, multiply each y-coordinate by -1.

$(x, y) \rightarrow (x, -y)$
$A(-4, 3) \rightarrow A'(-4, -3)$
$B(1, 3) \rightarrow B'(1, -3)$

$(x, y) \rightarrow (x, -y)$
$C(0, 1) \rightarrow C'(0, -1)$
$D(-5, 1) \rightarrow D'(-5, -1)$

The coordinates of the vertices of the image are $A'(-4, -3)$, $B'(1, -3)$, $C'(0, -1)$, and $D'(-5, -1)$.

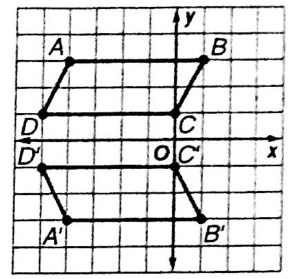
...udy Tip

...eading Math
...rallelogram *ABCD* and ...image *A'B'C'D'* are ...id to be **symmetric** ...out the *x*-axis. The ...axis is called the **line** ...**symmetry**.

b. Graph parallelogram $ABCD$ and its image $A'B'C'D'$.

Graph each vertex of the parallelogram $ABCD$. Connect the points.

Graph each vertex of the reflected image $A'B'C'D'$. Connect the points.



Example 3 Translation

Triangle ABC has vertices $A(-2, 3)$, $B(4, 0)$, and $C(2, -5)$.

a. Find the coordinates of the vertices of the image if it is translated 3 units to the left and 2 units down.

To translate the triangle 3 units to the left, add -3 to the x-coordinate of each vertex. To translate the triangle 2 units down, add -2 to the y-coordinate of each vertex.

$$(x, y) \rightarrow (x - 3, y - 2)$$

$$A(-2, 3) \rightarrow A'(-2 - 3, 3 - 2) \rightarrow A'(-5, 1)$$

$$B(4, 0) \rightarrow B'(4 - 3, 0 - 2) \rightarrow B'(1, -2)$$

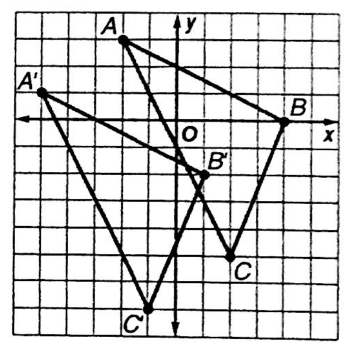
$$C(2, -5) \rightarrow C'(2 - 3, -5 - 2) \rightarrow C'(-1, -7)$$

The coordinates of the vertices of the image are $A'(-5, 1)$, $B'(1, -2)$, and $C'(-1, -7)$.

b. Graph triangle ABC and its image.

The preimage is $\triangle ABC$.

The translated image is $\triangle A'B'C'$.



Example 4 Dilation

A trapezoid has vertices $L(-4, 1)$, $M(1, 4)$, $N(7, 0)$, and $P(-3, -6)$.

a. Find the coordinates of the dilated trapezoid $L'M'N'P'$ if the scale factor is $\frac{3}{4}$.

To dilate the figure multiply the coordinates of each vertex by $\frac{3}{4}$.

$$(x, y) \rightarrow \left(\frac{3}{4}x, \frac{3}{4}y\right)$$

$$L(-4, 1) \rightarrow L'\left(\frac{3}{4} \cdot (-4), \frac{3}{4} \cdot 1\right) \rightarrow L'\left(-3, \frac{3}{4}\right)$$

$$M(1, 4) \rightarrow M'\left(\frac{3}{4} \cdot 1, \frac{3}{4} \cdot 4\right) \rightarrow M'\left(\frac{3}{4}, 3\right)$$

$$N(7, 0) \rightarrow N'\left(\frac{3}{4} \cdot 7, \frac{3}{4} \cdot 0\right) \rightarrow N'\left(5\frac{1}{4}, 0\right)$$

$$P(-3, -6) \rightarrow P'\left(\frac{3}{4} \cdot (-3), \frac{3}{4} \cdot (-6)\right) \rightarrow P'\left(-2\frac{1}{4}, -4\frac{1}{2}\right)$$

The coordinates of the vertices of the image are $L'\left(-3, \frac{3}{4}\right)$, $M'\left(\frac{3}{4}, 3\right)$, $N'\left(5\frac{1}{4}, 0\right)$, and $P'\left(-2\frac{1}{4}, -4\frac{1}{2}\right)$.

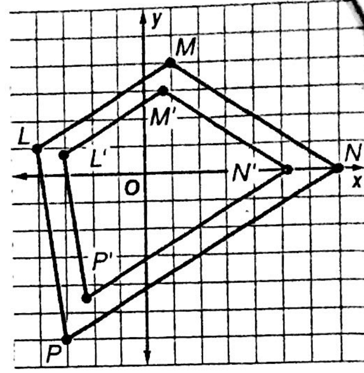
(continued on the next page)

b. Graph the preimage and its image.

The preimage is trapezoid *LMNP*.

The image is trapezoid *L'M'N'P'*.

Notice that the image has sides that are three-fourths the length of the sides of the original figure.



Example 5 Rotation

Triangle *XYZ* has vertices *X*(1, 5), *Y*(5, 2), and *Z*(−1, 2).

a. Find the coordinates of the image of △*XYZ* after it is rotated 90° counterclockwise about the origin.

To find the coordinates of the vertices after a 90° rotation, switch the coordinates of each point and then multiply the new first coordinate by −1.

$(x, y) \rightarrow (-y, x)$

$X(1, 5) \rightarrow X'(-5, 1)$

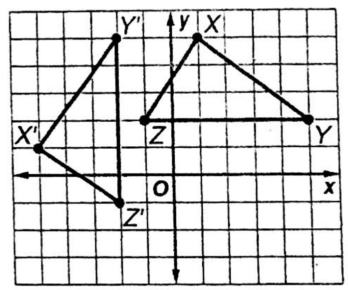
$Y(5, 2) \rightarrow Y'(-2, 5)$

$Z(-1, 2) \rightarrow Z'(-2, -1)$

b. Graph the preimage and its image.

The image is △XYZ.

The rotated image is △X'Y'Z'.



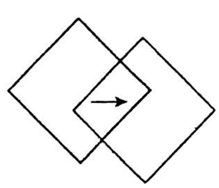
Check for Understanding

Concept Check

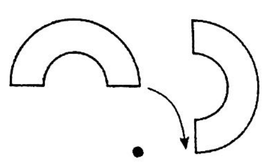
1. **Compare and contrast** the size, shape, and orientation of a preimage and an image for each type of transformation.
2. **OPEN ENDED** Draw a figure on the coordinate plane. Then show a dilation of the object that is an enlargement and a dilation of the object that is a reduction.

Guided Practice **Identify each transformation as a *reflection, translation, dilation,* or *rotation*.**

3.



4.



Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image.

5. triangle *PQR* with *P*(1, 2), *Q*(4, 4), and *R*(2, −3) reflected over the *x*-axis
6. quadrilateral *ABCD* with *A*(4, 2), *B*(4, −2), C(−1, −3), and *D*(−3, 2) translated 3 units up
7. parallelogram *EFGH* with *E*(−1, 4), *F*(5, −1), *G*(2, −4), and *H*(−4, 1) dilated by a scale factor of 2
8. triangle *JKL* with *J*(0, 0), *K*(−2, −5), and *L*(−4, 5) rotated 90° counterclockwise about the origin

Application **NAVIGATION** **For Exercises 9 and 10, use the following information.**
A ship was heading on a chartered route when it was blown off course by a storm. The ship is now ten miles west and seven miles south of its original destination.

9. Using a coordinate grid, make a drawing to show the original destination A and the current position B of the ship.

10. Using coordinates (x, y) to represent the original destination of the ship, write an ordered pair to show its current location.

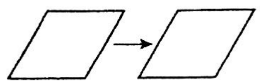
Practice and Apply

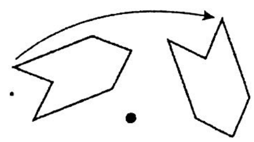
Homework Help

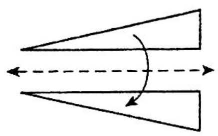
For Exercises	See Examples
11–16, 37, 38	1
17–36	2–5

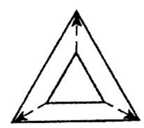
Extra Practice
See page 828.

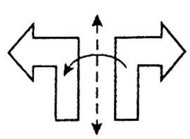
Identify each transformation as a *reflection, translation, dilation,* or *rotation.*

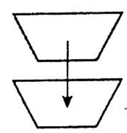
11. 

12. 

13. 

14. 

15. 

16. 

For Exercises 17–26, complete parts a and b.

a. **Find the coordinates of the vertices of each figure after the given transformation is performed.**

b. **Graph the preimage and its image.**

17. triangle RST with $R(2, 0)$, $S(-2, -3)$, and $T(-2, 3)$ reflected over the y-axis

18. trapezoid $ABCD$ with $A(2, 3)$, $B(5, 3)$, $C(6, 1)$, and $D(-2, 1)$ reflected over the x-axis

19. quadrilateral $RSTU$ with $R(-6, 3)$, $S(-4, 2)$, $T(-1, 5)$, and $U(-3, 7)$ translated 8 units right

20. parallelogram $MNOP$ with $M(-6, 0)$, $N(-4, 3)$, $O(-1, 3)$, and $P(-3, 0)$ translated 3 units right and 2 units down

21. trapezoid $JKLM$ with $J(-4, 2)$, $K(-2, 4)$, $L(4, 4)$, and $M(-4, -4)$ dilated by a scale factor of $\frac{1}{2}$

22. square $ABCD$ with $A(-2, 1)$, $B(2, 2)$, $C(3, -2)$, and $D(-1, -3)$ dilated by a scale factor of 3

23. triangle FGH with $F(-3, 2)$, $G(2, 5)$, and $H(6, 3)$ rotated 180° about the origin

24. quadrilateral $TUVW$ with $T(-4, 2)$, $U(-2, 4)$, $V(0, 2)$, and $W(-2, -4)$ rotated 90° counterclockwise about the origin

25. parallelogram $WXYZ$ with $W(-1, 2)$, $X(3, 2)$, $Y(0, -4)$, and $Z(-4, -4)$ reflected over the y-axis, then rotated 180° about the origin

26. pentagon $PQRST$ with $P(0, 5)$, $Q(3, 4)$, $R(2, 1)$, $S(-2, 1)$, and $T(-3, 4)$ reflected over the x-axis, then translated 2 units left and 1 unit up